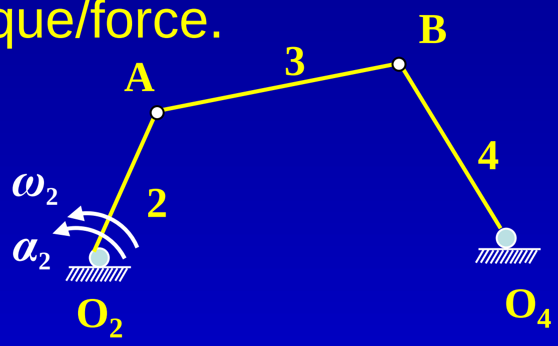
Force Analysis

Kinetic analysis has to be performed to design the shape and thickness of the links, the joints and to determine the required input torque/force and the resulting output torque/force.



Force analysis of a 4-Bar

Known:

- The length of all links; r_1, r_2, r_3, and r_4
- Input information; θ_2, ω_2, α_2, and T_2 (torque)
- The results of position, velocity, and acceleration analysis; θ_3, θ_4, ω_3, ω_4, and α_3, α_4
- All external loads acting on the links

Unknown:

- Forces acting on all joints; F_{O_2}, F_A, F_B, and F_{O_4}
- Inertia forces acting on all links; F_{I2}, F_{I3}, F_{I4},
- Inertia torques acting on all links; T_{I2}, T_{I3}, T_{I4},
- The input torque (or force) needed to obtain the required output torque (or force)

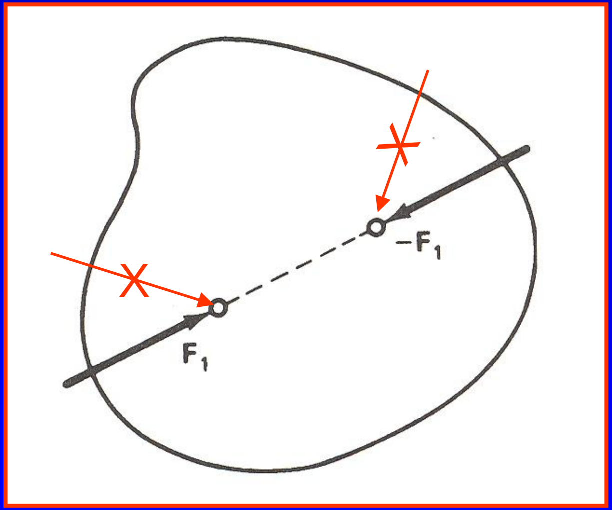
Force Analysis – Static Equilibrium

Static equilibrium – 2D case

$$\sum F_x = 0, \ \sum F_y = 0, \ \sum M = 0$$

Two force members – only two forces acting, one at each joint.

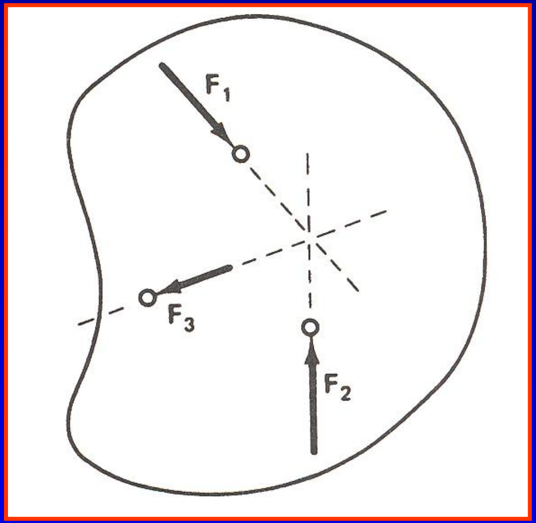
A rigid link acted on by two forces is in static equilibrium only if the two forces are collinear and have the same magnitude in opposite direction



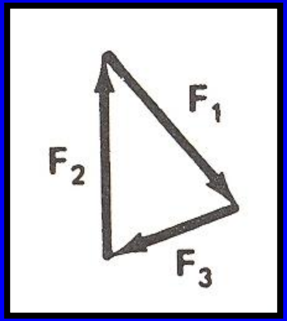
$$\overline{F}_1 = -\overline{F}_2$$

Force Analysis – Static Equilibrium

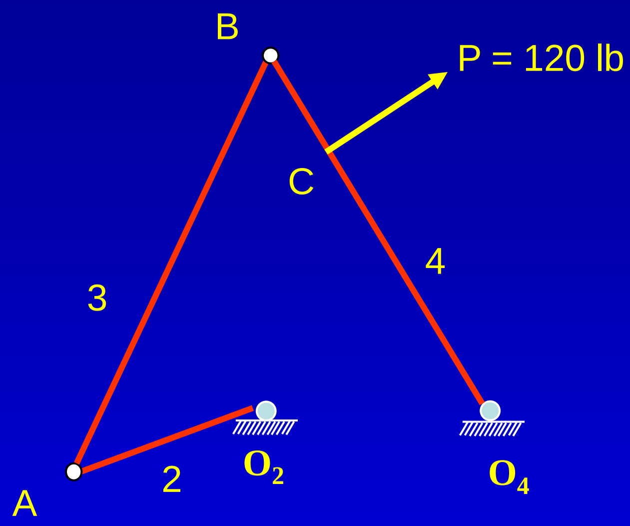
Three force members – only three forces act on a link. Either three joint forces, or two joint forces and an external or inertia or gravitational force.



The link is in static equilibrium only if the force-vectors add up to zero and they pass through the same point (summation of moment about that point is zero).



Force Analysis – Graphical Method

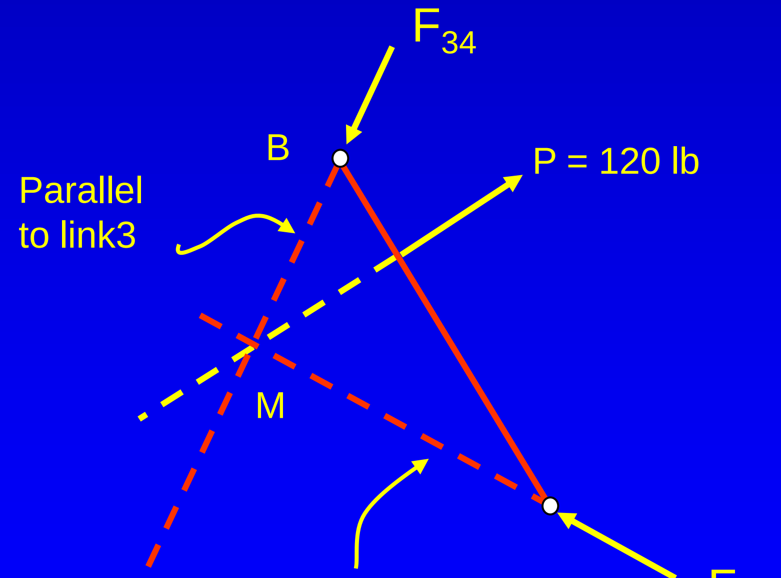


$O_2A = 3$, $AB = O_4B = 6$, $O_4C = 4.5$,
$O_2O_4 = 2.4$

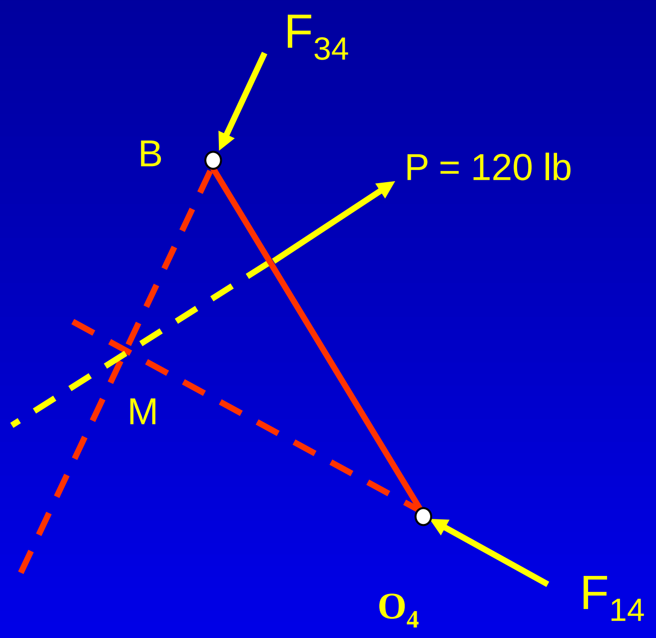
Find the input torque, T_2 , to maintain static equilibrium at the position shown or what should be the input torque to exert 120 lb of force for the position shown. Use graphical method, construct force polygon.

Free-Body Diagram of link 4

AB is a two force member, F_{34} (force of link3 on link 4) has to be collinear to link 3.



Force Analysis – Graphical Method

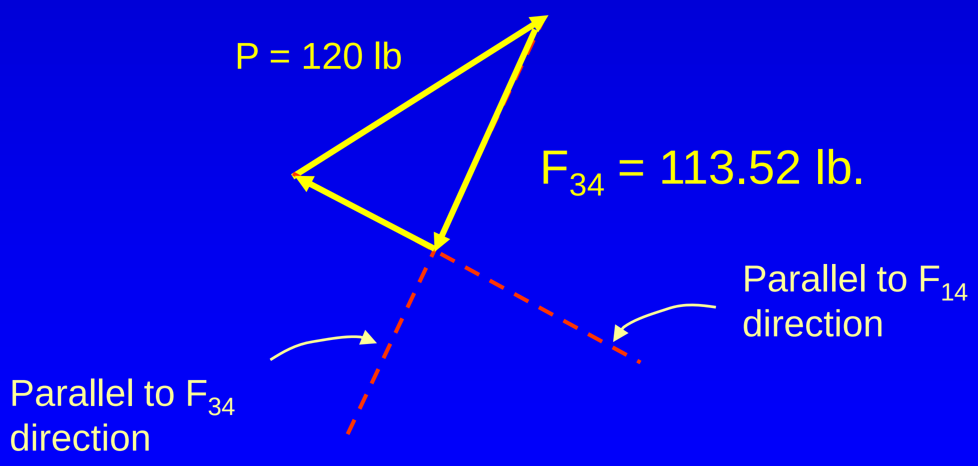


Force polygon

1 - Draw and scale force P

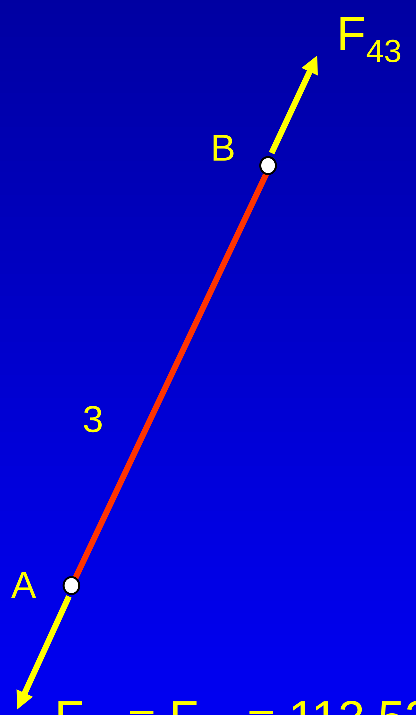
2 - Draw two lines parallel to F_{14} and F_{34} directions from the two ends of vector P

3 - Locate the intersection, complete the polygon and scale the force F_{34}



Force Analysis – Graphical Method

Free-Body Diagram of link 3

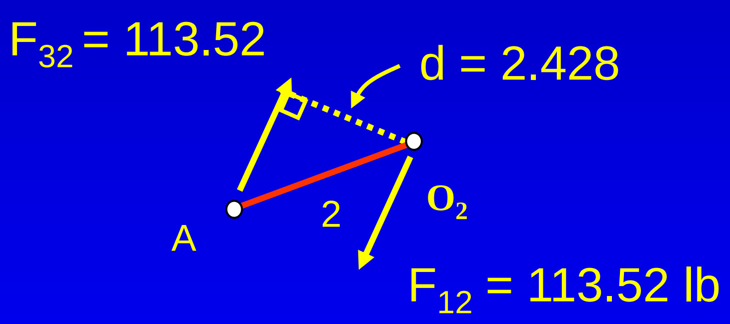


$F_{23} = F_{43} = 113.52$

Free-Body Diagram of link 2

Draw and scale link 2, $O_2A = 3$

Scale the perpendicular distance from O_2 to the force F_{32}

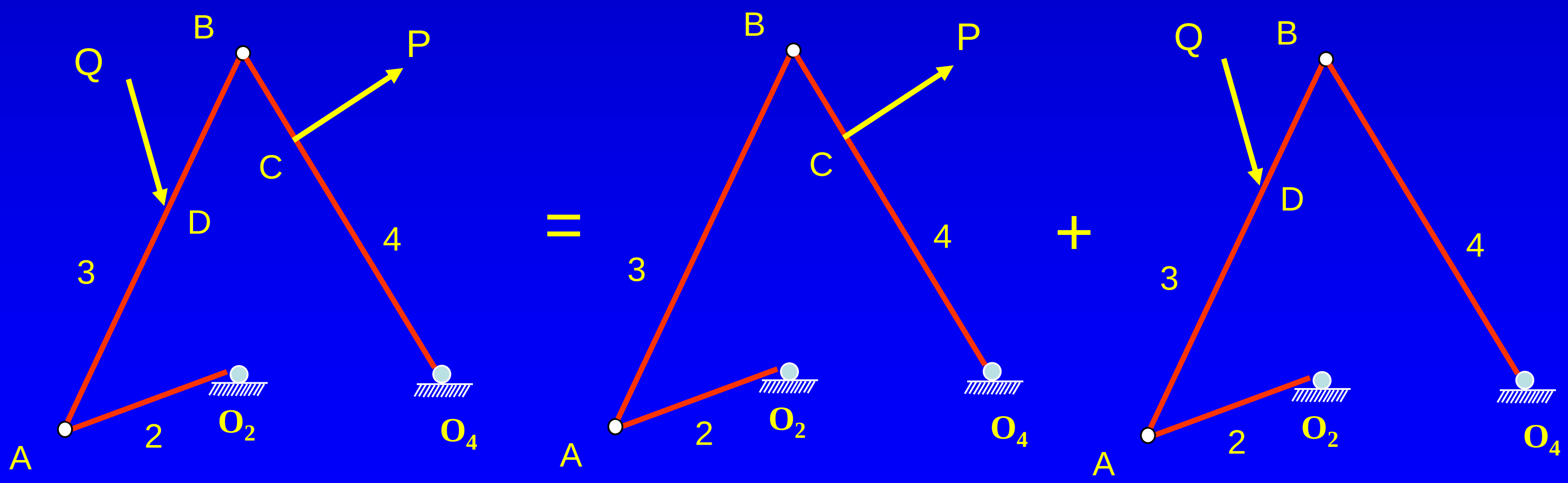


$T_2 = (F_{32})(d) = 113.52 \times 2.428 = 275.6$ in-lb (ccw)

Force Analysis – Graphical Method
Principle of Superposition

In linear systems the output (response) is directly proportional to the input, linear relationship between the output and the input exist.

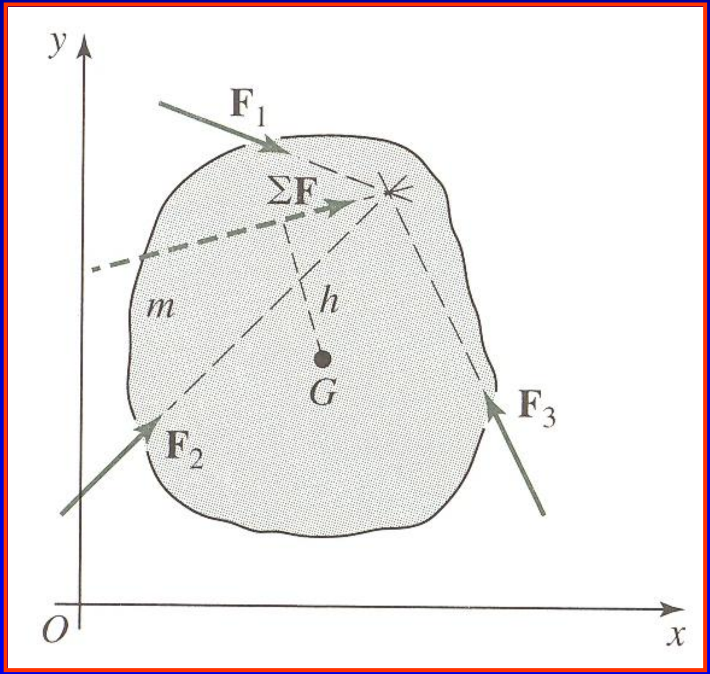
Principle of superposition can be used to solve problems involving linear systems by considering each of the inputs to the system separately. And then, combine the individual results to obtain the total response of the system.



$$\overline{T}_2 = (\overline{T}_2)_{\text{Due to force P}} + (\overline{T}_2)_{\text{Due to force Q}}$$

Dynamic Force Analysis

Inertia forces and D'Alembert's Principle



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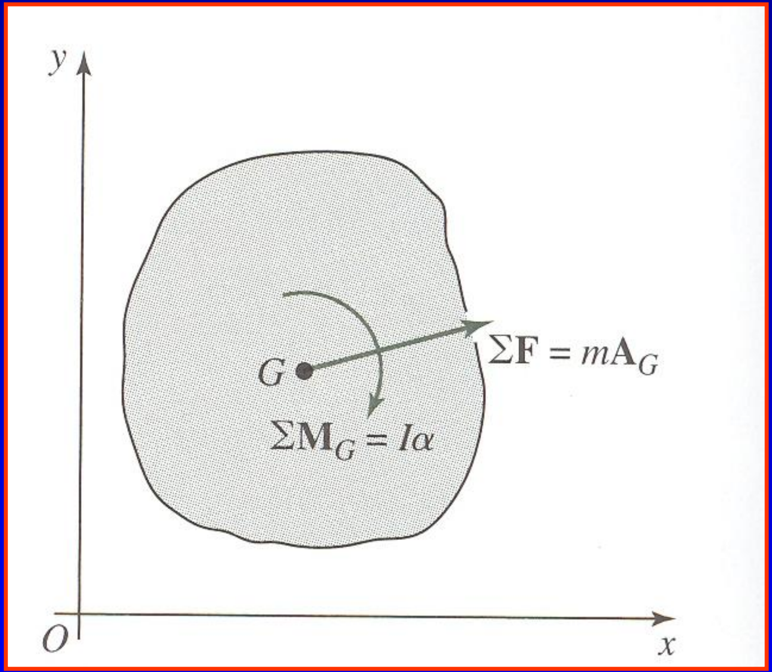


An unbalanced set of forces acting on a rigid body

The translation and rotation caused by the unbalanced forces

Resultant force **R** = ∑ **F**

R = (mass)(linear acceleration of the center of gravity), R = (m)$\mathbf{a}_G$

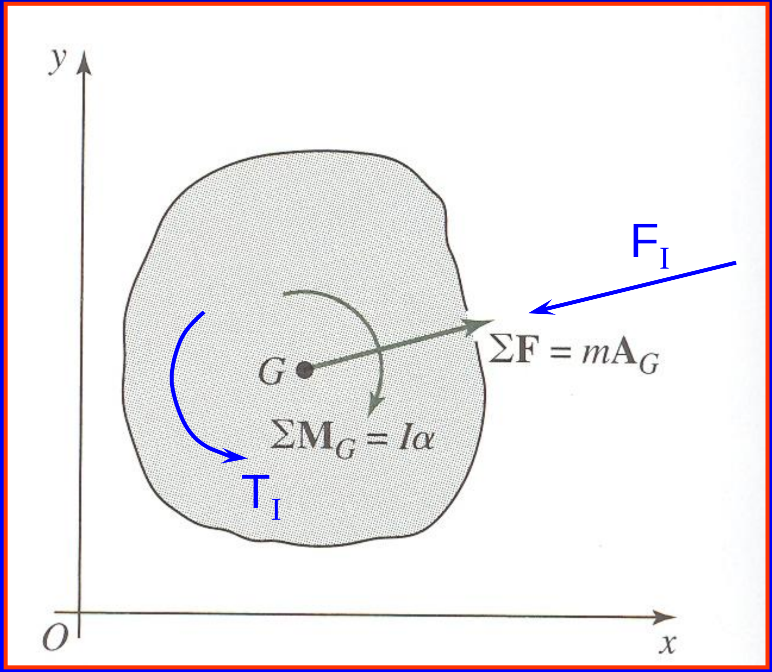
$\mathbf{M}_G$ = (area moment of inertia)(angular acceleration) = $I_G\alpha$ = (R)(h)

Dynamic Force Analysis

D'Alembert's Principle

$R - (m)a_G = 0$, inertia force = $F_I = -ma_G$

$\mathbf{M_G} - I_G\alpha = 0$, inertia torque = $T_I = -I_G\alpha = -Rh = F_I h$



The ***inertia force*** has magnitude of $\boldsymbol{ma_G}$ and it acts in the opposite direction of linear acceleration. It stops motion.

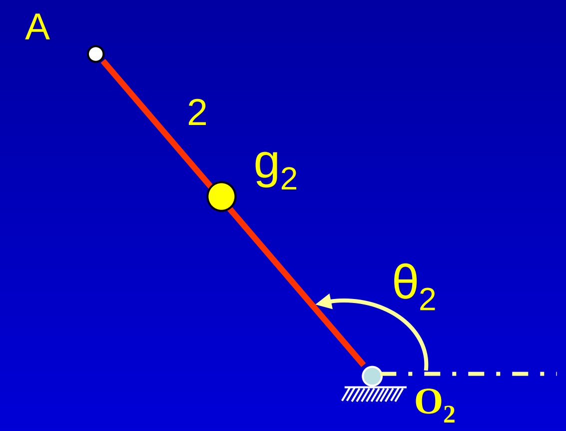
The ***inertia torque*** has a magnitude of $\boldsymbol{(F_I)(h)}$ and it acts in the opposite direction of angular acceleration.

Inertia Force and Torque - Example

Input info: $\theta_2 = 144^o$, $\omega_2 = 12$ rad/sec (ccw), $\alpha_2 = 60$ rad/sec^2 (ccw)

Geometry: $I_2 = .04$ lb-ft-sec^2, $W_2 = 6$ lb.

$O_2A = 12$ in., $O_2g_2 = 6.6$ in.



Determine the inertia force and the inertia torque of link 2, magnitude and direction. Show the force and the torque on the link.

First calculate the linear acceleration of the center of gravity of link 2, a_{g_2}

Position of g_2, $\overline{r}_{g_2} = r_{g_2}\, e^{i\theta_2}$

Velocity of g_2, $\overline{V}_{g_2} = r_{g_2}\, \omega_2\, i\, e^{i\theta_2}$

Acceleration of g_2, $\overline{a}_{g_2} = r_{g_2}\, \alpha_2\, i\, e^{i\theta_2} - r_{g_2}\, (\omega_2)^2\, e^{i\theta_2}$

Inertia Force and Torque - Example

$$\overline{a_{g_2}} = r_{g_2}\, e^{i\,\theta_2}\, [\, \alpha_2\, i - (\omega_2)^2\,]$$

$$\overline{a_{g_2}} = 6.6\, e^{i\,(144)}\, [\, 60\, i - (12)^2\,]$$

$$\overline{a_{g_2}} = 6.6\, [\, \cos(144) + i\, \sin(144)\,]\, (60\, i - 144)$$

$$\overline{a_{g_2}} = 536.1 - 879\, i$$

$$a_{g_2} = \sqrt{(536.1)^2 + (879)^2} = 1029.6 \text{ in/sec}^2$$

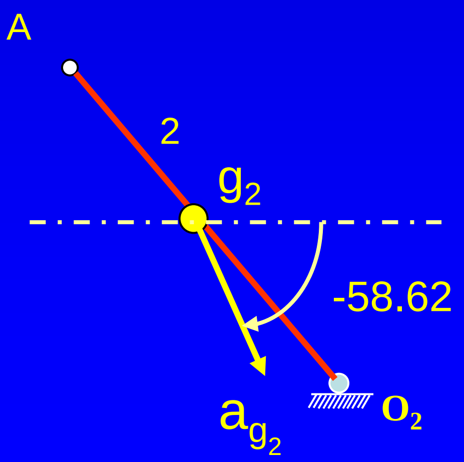
$$\beta_2 = \tan^{-1}(-879/536.1) = -58.62$$

Inertia force of link 2 = $F_{I\,2} = m_2\, a_{g_2}$

$$F_{I\,2} = (6/386)(1029.6) = 16 \text{ lb}$$

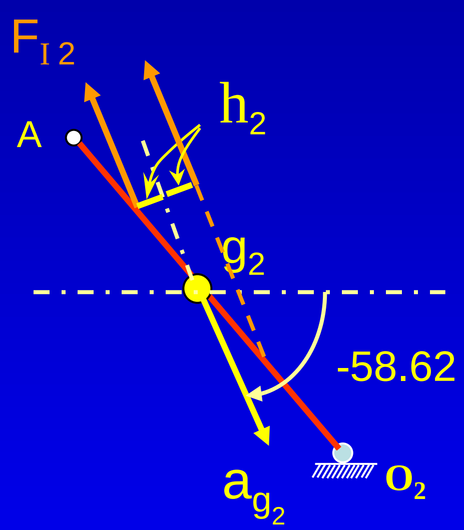
$$h_2 = \frac{I_2\, \alpha_2}{F_{I\,2}} = \frac{.04 \times 12 \times 60}{16} = 1.8\text{ “}$$

$$T_{I\,2} = F_{I\,2} \times h_2 = 16 \times 1.8 = 28.8 \text{ in-lb}$$

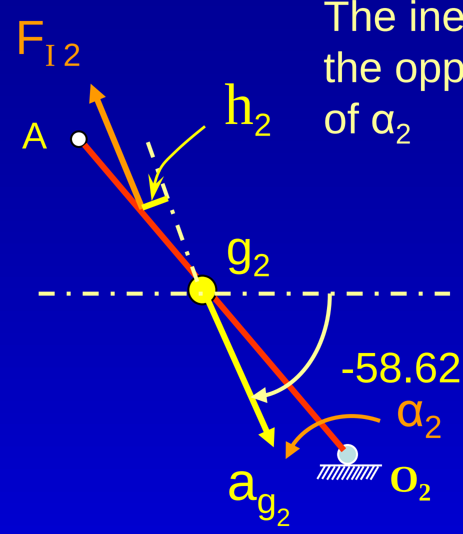


Inertia Force and Torque - Example

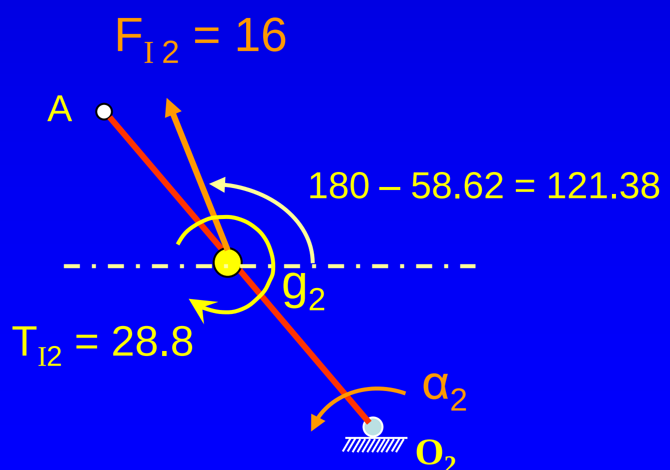
The inertia force is in the opposite direction of the acceleration of the center of gravity



The inertia torque is in the opposite direction of α_2

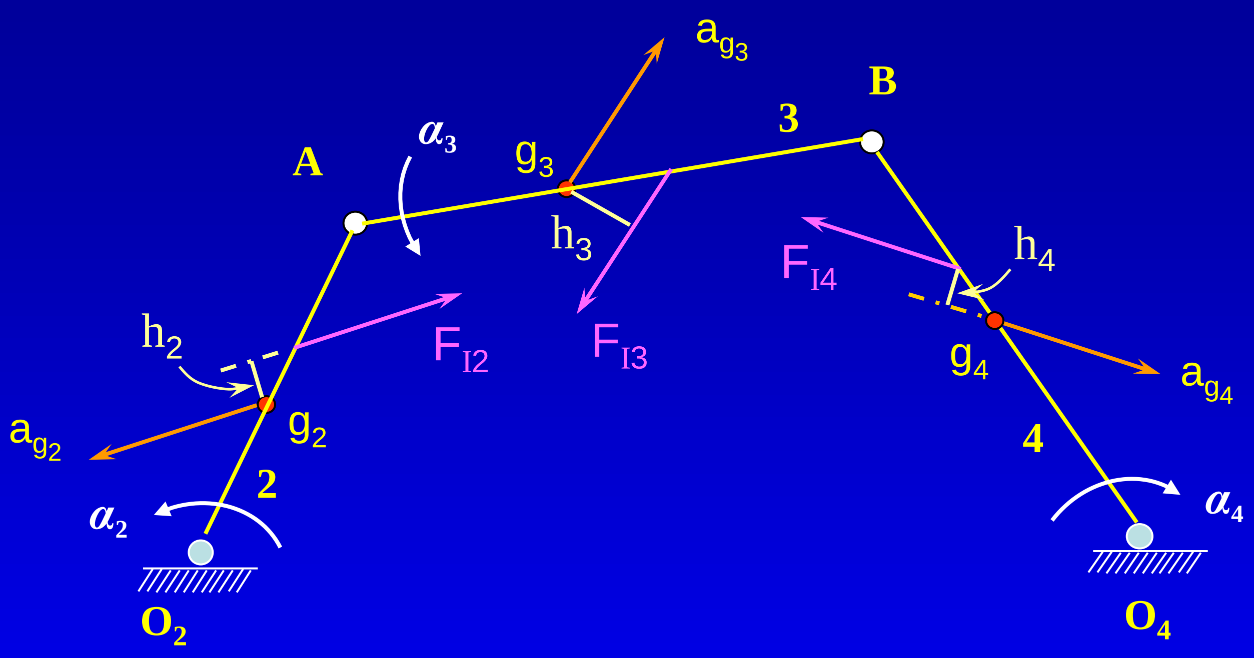


Inertia force an torque acting on link 2



Force Analysis – Analytical, Matrix Method

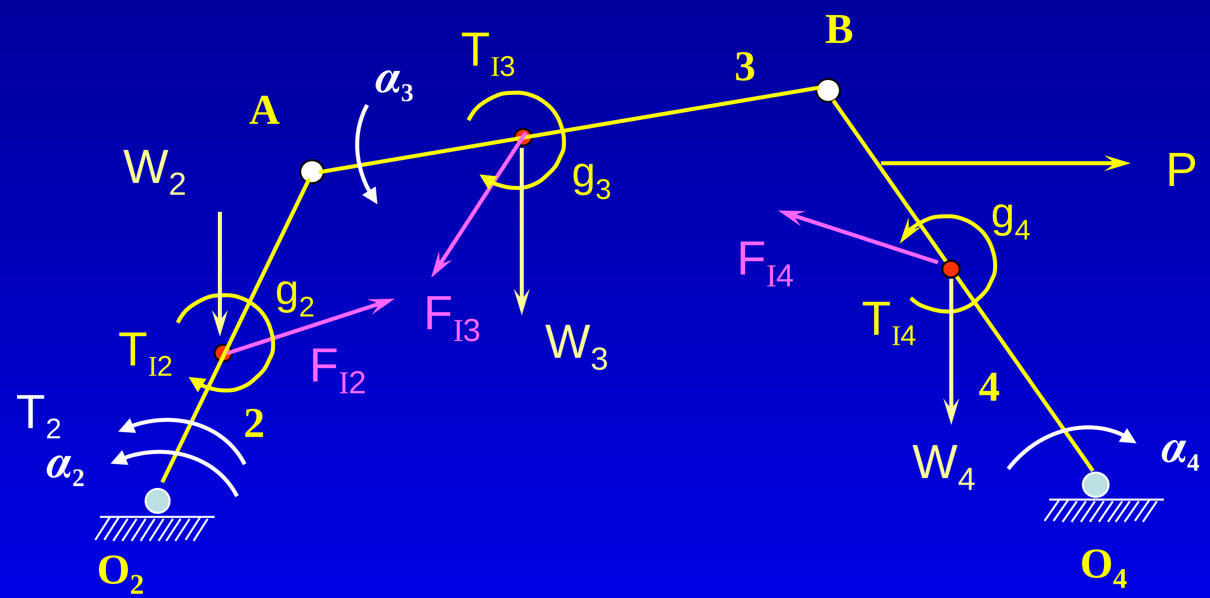
Four bar mechanism



	Link 2	Link 3	Link 4
Inertia force	$F_{I2} = m_2\, a_{g_2}$	$F_{I3} = m_3\, a_{g_3}$	$F_{I4} = m_4\, a_{g_4}$
Torque arm	$h_2 = (I_2\alpha_2)/F_{I2}$	$h_3 = (I_3\alpha_3)/F_{I3}$	$h_4 = (I_4\alpha_4)/F_{I4}$
Inertia torque	$T_{I2} = F_{I2}\, h_2$ cw	$T_{I3} = F_{I3}\, h_3$ cw	$T_{I4} = F_{I4}\, h_4$ ccw

Force Analysis – Analytical, Matrix Method

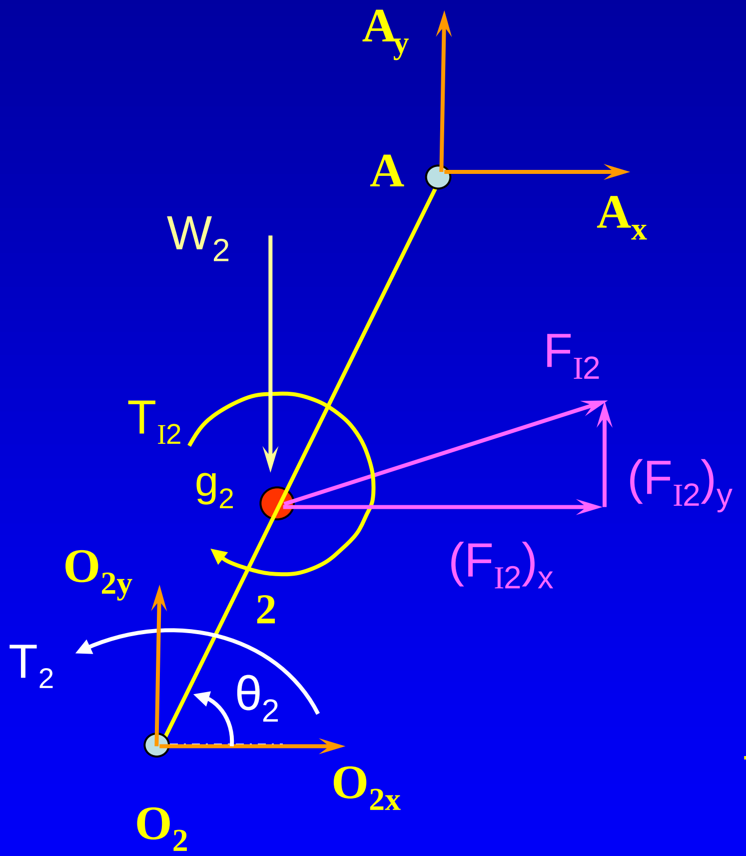
Show all inertia forces and torques on the four bar mechanism



Show all external and gravitational forces,
location and magnitude of force P is given.

Force Analysis – Analytical, Matrix Method

Free body diagram of link 2. Assume the direction of forces at joints A and O_2



$O_2A = r_2$, $O_2g_2 = r_{g_2}$

$$\sum F_x = 0$$

$$A_x + O_{2x} + (F_{I2})_x = 0 \qquad (1)$$

$$\sum F_y = 0$$

$$A_y + O_{2y} + (F_{I2})_y - W_2 = 0 \qquad (2)$$

$$\sum M_g = 0$$

$$- A_x (r_2 - r_{g_2})\sin\theta_2 + A_y (r_2 - r_{g_2})\cos\theta_2$$

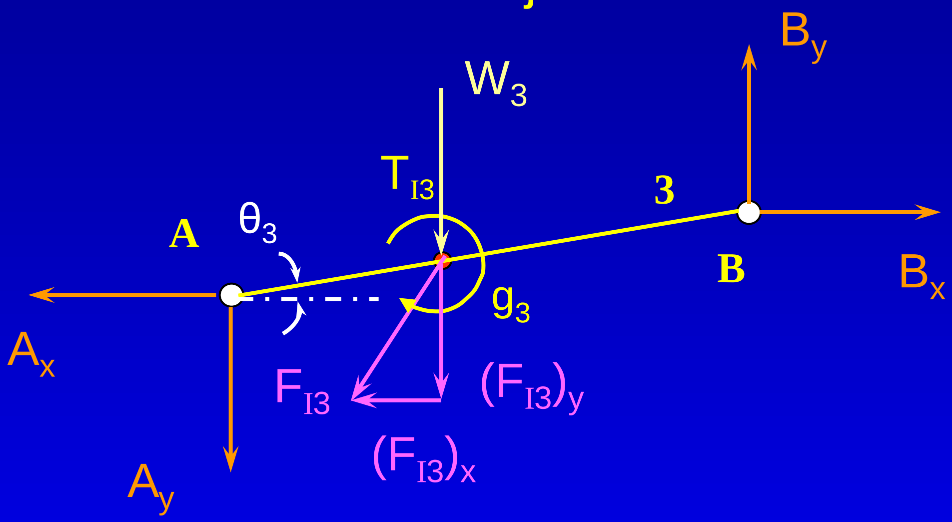
$$+ O_{2x} (r_{g_2})\sin\theta_2 - O_{2y} (r_{g_2})\cos\theta_2 + T_2 - T_{I2} = 0$$

$$(3)$$

$$- A_x C_1 + A_y C_2 + O_{2x} C_3 - O_{2y} C_4 + T_2 - T_{I2} = 0$$

Force Analysis – Analytical, Matrix Method

Free body diagram of link 3. Direction of the force at A has to be opposite of the direction assumed on link 2. Assume the direction of the force at joint B.



$\sum F_x = 0$

$$-A_x + B_x - (F_{I3})_x = 0 \qquad (4)$$

$\sum F_y = 0$

$$-A_y + B_y - (F_{I3})_y - W_3 = 0 \qquad (5)$$

$\sum M_g = 0$

$AB = r_3, \ Ag_3 = r_{g_3}$

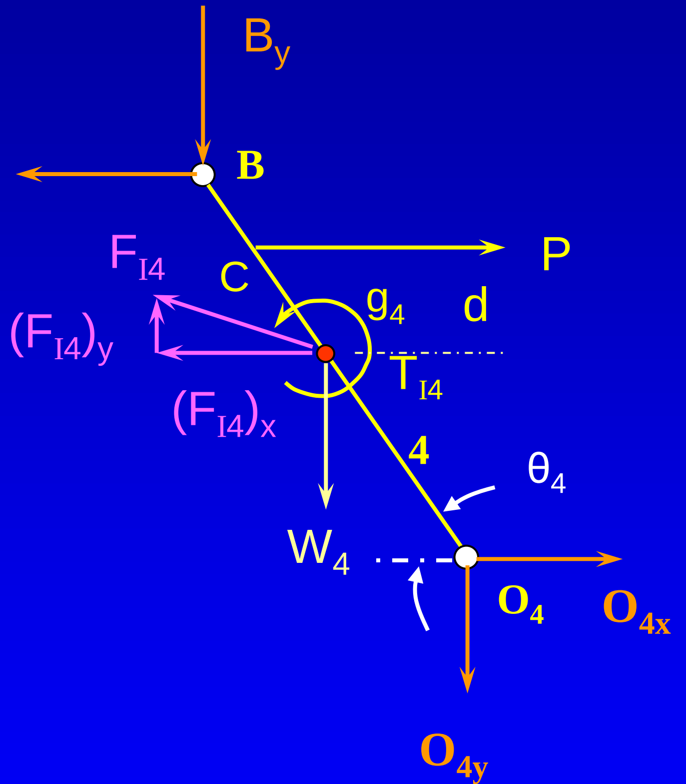
$$-B_x (r_3 - r_{g_3})\sin\theta_3 + B_y (r_3 - r_{g_3})\cos\theta_3$$

$$-A_x (r_{g_3})\sin\theta_3 + A_y (r_{g_3})\cos\theta_3 - T_{I3} = 0$$

(6)

Force Analysis – Analytical, Matrix Method

Free body diagram of link 4. Direction of the force at B has to be opposite of the direction assumed on link 3. Assume the direction of the force at joint O_4.



$\sum F_x = 0$

$$+ O_{4x} - B_x - (F_{I4})_x + P = 0 \qquad (7)$$

$\sum F_y = 0$

$$- O_{4y} - B_y + (F_{I4})_y - W_4 = 0 \qquad (8)$$

$\sum M_g = 0$

$$B_x (r_4 - r_{g_4})\sin\theta_4 + B_y (r_4 - r_{g_4})\cos\theta_4$$

$$+ O_{4x} (r_{g_4})\sin\theta_4 - O_{4y} (r_{g_4})\cos\theta_4 + T_{I4} - pd = 0$$

(9)

$O_4B = r_4, \; O_4g_4 = r_{g_4}$

$d = (O_4C - r_{g_4})\sin\theta_4$